



Math Lab Lesson #4: Working with Fractions

How can we interpret the fraction $\frac{a}{b}$?

How can we multiply and divide fractions?

SITUATING THE LESSON:

During Saturday Academy and Weekday, students have:

- Explored four different ways of understanding fractions: the part/whole model; the partitive (sharing) model; the quotative (measuring) model; and the number model.
- Compared the relative magnitudes of different fractions by finding a common denominator and comparing numerators
- Solved simple linear equations with fractional coefficients by finding a common denominator and operating with numerators.

Summary: In this lesson, students will review some of the key ideas that they have learned during Saturday Academy Lesson 6. The main goal of this lesson was to introduce four different models of thinking about fractions, and explore the benefits and drawbacks of each (see Background Information). **Also, make sure to get to the Group Activity!**

Working with fractions is often difficult for students precisely because there are many correct ways of interpreting what $\frac{a}{b}$ “means,” and these interpretations depend on context. Part of mathematical fluency is developing flexibility in interpreting the same written symbol in multiple ways. This week, we will: review these four different models; explore the benefits of finding a common denominator in a variety of contexts; use the “measuring” model to explore situations that require iterative thinking (i.e. “how many lengths x fit into a length y ?”); and begin to operate on numbers (multiply/divide).

Preparation Before Class: Work through all problems and watch videos in advance. Read through and annotate the Lesson Plan in a way that will be useful to you.

Materials:

- **Math Lab Lesson #4: Classwork** (1 per student and instructor)
- **Math Lab Lesson #4: Group Activity** (1 per student and instructor)
- **Twine, Tape (Scotch or masking), Scissors, and Tape Measures (10’)**
- **Math Lab Lesson #4: Exit Ticket** (1 per student and instructor)
- **Math Lab Lesson #4: Instructor Answer Key** (instructor only)

Background Information (for Instructor only)

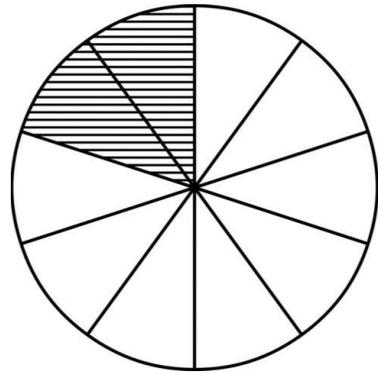
In Saturday Academy Lesson 6, we went over the following:

Four Different Ways of Understanding Fractions

1. **Part/Whole Model:** The fraction $\frac{2}{10}$ means “start with one whole unit (like a pie), divide it into 10 **equal-sized** parts, then take 2 of these parts. This is the picture in your mind:

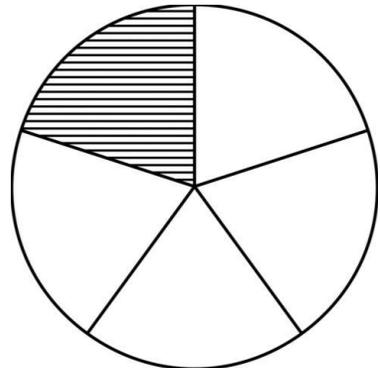
Benefit #1: Viewing a fraction as a $\frac{\text{part}}{\text{whole}}$ comparison is the basis of percents (%). When calculating percents, the whole is always 100. For example, 20% of the circle is shaded because:

$$\frac{2}{10} = \frac{20}{100}$$



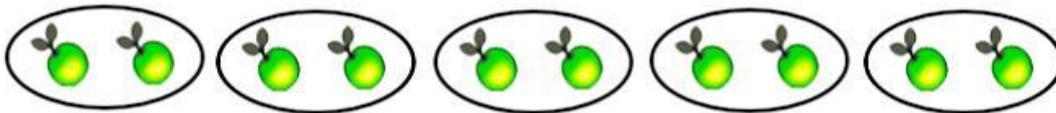
Benefit #2: Proportions make sense For example, we can see from the 2nd pie picture that:

$$\frac{2}{10} = \frac{1}{5}$$



Limitation: We can't divide by a fraction with this model. What would $\frac{1}{\left(\frac{1}{5}\right)}$ mean? The model breaks down.

2. **Partitive (Sharing) Model:** The fraction $\frac{10}{5}$ means “Start with 10 objects (like apples), and share them **equally** among your 5 friends. Everyone gets 2 apples”



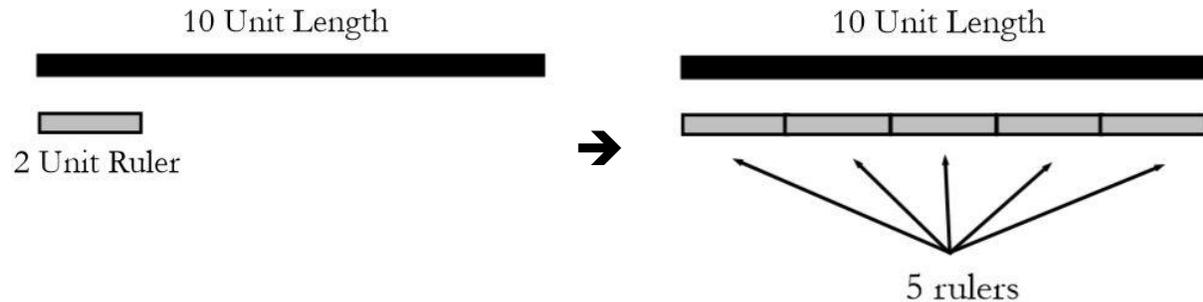
Benefit: With this model, there is a clear connection between multiplication and division. For example “If I have 5 friend and each friend has 2 apples, then there are $5 \times 2 = 10$ apples total.

Limitation: We can't divide by a fraction with this model. What does the fraction $\frac{10}{\left(\frac{1}{2}\right)}$ mean?

The model breaks down.

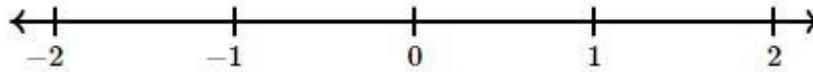


3. Quotative (Measuring) Model: The fraction $\frac{10}{2}$ means “Start with a length of 10 Units (like feet), and a ruler with a length of 2 Units (like feet). It will take 5 rulers to measure this length.”



Benefit: Everything! **Limitation:** Nothing!

4. The Number Model: It’s just a number! The fraction $\frac{5}{6}$ (“five sixths”) is a **rational number** that is equal to $5\left(\frac{1}{6}\right)$ or “Five of these things that are one-sixths.” Since it is a number, it is located on the number line.



1st Hour

1. Lesson Launch (15 min)

A) SATURDAY ACADEMY AND WEEKDAY REVIEW (5 MIN)

- Ask students what they have been learning in Saturday Academy and Weekday. What has been interesting? What have they struggled with? Take a few responses and jot them on the board.
- Ask students to get out their Workbook and find a Classwork (Level 1 or Level 2) problem that they understand. Have them pair up and give them one minute to explain it to their partner, then switch.

B) INTRODUCTION TO FRACTIONS (10 MIN)

- Write the fraction $\frac{2}{6}$ on the board, and ask students to take a minute and write down a possible situation which could be modeled by this fraction. Then share out and lead a discussion



about different ways of interpreting this fraction. Have students share their ideas and write the pictures/situations that they have on the board. Compare them, talk through them. It is okay if this takes a little longer than anticipated, think of this as a way to unearth some of the misunderstandings of middle school.

- There are multiple possibilities, for example:
 - Part-Whole Model: 1 pizza cut into 6 slices, you take two of these slices ($\frac{1}{3}$ rd of the pizza). This is similar to interpreting $\frac{2}{6} = 2 \cdot \left(\frac{1}{6}\right)$, in other words “two of these things that are one-sixth.” (Number Model)
 - Sharing Model: 2 apples shared among 6 people, how much does each person get? ($\frac{1}{3}$ rd of an apple)
 - Measuring Model: How much of a 6-foot ruler would it take to measure a 2-foot stick? ($\frac{1}{3}$ rd of the ruler)
- Reiterate that if students feel that fractions are confusing, that is an entirely legitimate way to feel. Fractions are confusing! The big confusion often comes about when considering what the “whole” is in a particular context. For example, in the first two possibilities above, the “whole” can be considered to be “1” (pizza) or “2” (apples). Both are reasonable interpretations. When you write the number “3” on the board, there is only one real way of interpreting it (discretely as a count of 3 things, or continuously as a 3 unit length). But with fractions, things get more complicated.
- Explain that it is often not useful to think of $\frac{2}{6}$ in a visual way, i.e. “2 over 6” or $\frac{\blacksquare\blacksquare}{\blacksquare\blacksquare\blacksquare\blacksquare\blacksquare\blacksquare}$, because the numbers themselves are not important, it’s the relationship between the numbers (the “ratio” of the numbers) that is at the heart of understanding fractions. That is why there are many different ways to write the rational number $\frac{2}{6} : \frac{2}{6} = \frac{1}{3} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15} = \frac{x}{3x} \dots$. The actual values of the numerator and denominator don’t matter, it’s the fact that the denominator is three times as large as the numerator that is the important part. This is the fundamental observation that gives students access to proportional reasoning and the slope concept.



2. Individual Work: Math Lab Videos (45 min)

- Hand out the Classwork and help students access the following videos:
 - Video #1: Four Fraction Models and Common Denominators (what are they good for?)
 - Video #2: Finding a Common Denominator in Different Contexts
 - Video #3: The Measuring Model in Different Contexts
 - Video #4: Introduction to Operations with Fractions
- Direct students to take notes on the videos on their Classwork. Let them know that they should feel free to pause or rewind the video if anything is unclear, and they should raise their hand if they would like you to help them one-on-one.
- Once a student is done watching a video, they should **complete the Classwork problems that relate to that video**. After they do this (and ask any clarifying questions), they should start the next video.
- As you circulate, make sure that students are focused and on track (i.e. not surfing the web), and provide hints and help as needed.
- NOTE: It is okay if this part of the lesson takes longer than planned and spills into the 2nd Hour. Students may go at different paces. If some students finish before other students, feel free to have them discuss their answers with each other, then start the Group Activity in pairs or groups of three.



2nd Hour

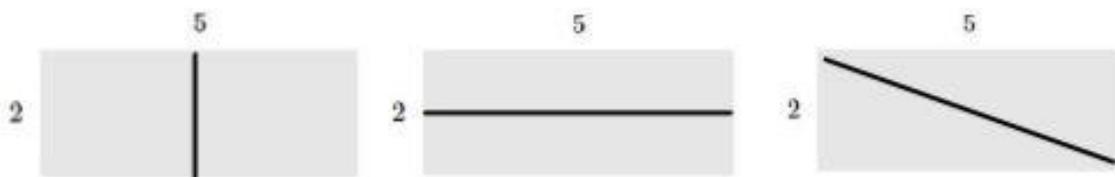
3. Break (5 min)

4. Whole Class Share-Out (5 min)

- Ask students “what did you learn from these videos?” and have a few students share out. Be very supportive while also encouraging students to use precise mathematical language.
- Give students a few minutes to check their answers with each other, then go over any outstanding questions with the class. Encourage students to present/explain their work.

5. Group Activity and Presentations (30 min)

- Hand out the Group Activity, and distribute twine, scotch tape and tape measures as necessary. Have students work on this activity groups of three or four. Circulate and provide support.
- The first few problems are all about dividing the table into $\frac{1}{2}$ and $\frac{1}{4}$ in multiple ways. The table itself is 2 ft \times 5 ft. Students can divide the table in half in a few different ways, i.e.:



- Quarters follow from halves.
- When students need them, hand out small flashcards as well. One important part is for students to use the measuring model explicitly to measure the length and width of the small flashcards in feet as $\frac{1}{6}$ ft and $\frac{1}{4}$ ft (2 inches and 3 inches). The big payoff is for students to calculate how many small cards would cover the entire table (without actually doing it). The dimensions of the flash card allow students to measure by having the longer side of the flashcard be either parallel or perpendicular with the longer side of the table. If students do it one way, ask them “would your answer change if you measured in the other way? Why or why not?” This prompt may help students develop a deeper understanding of area conservation.



- Once everyone is done, have groups present their answers and reasoning. Make connections between students' work. Clarify key ideas/concepts.
- If students finish earlier, have them try to divide the table into thirds without measuring (impossible I think, like trisecting an angle). Then, allow them to take out their Workbooks and do a few problems from the Classwork that they have not done yet.

6. Flashcards (10 min)

- Have students create 5 flashcards from what they learned today.
- Explain that you can create and use flashcards to study math concepts in the same way that you use them to learn vocabulary. You can write examples, key ideas, or even pictures (i.e. the “area model of multiplication”)
- **If necessary, model how to study with flashcards.** For example, introduce one flashcard at a time, mix them up as you study, etc.

7. Exit Ticket (5 min)

- If there is time left, hand out the Exit Ticket. Give students 3 minutes to work on it on their own silently, and then have them share out. Clear up any misconceptions, and dismiss the class.